

# HYBRID MODE COUPLING OF DIELECTRIC RESONATORS TO MICROSTRIP LINES

Xiao-Peng Liang and Kawthar A. Zaki

Electrical Engineering Department, University Of Maryland  
College Park, Maryland 20742

## ABSTRACT

Coupling between hybrid modes in dielectric resonators and microstrip lines is analyzed. This coupling is due to both magnetic and electric fields. These two kinds of coupling can be treated separately. Two degenerate hybrid modes with the same resonant frequency are excited by magnetic coupling and electric coupling, respectively. Computed data shows dependence of the external  $Q$  on the distance between the resonator and the line. Experimental measurements are compared with the computed data, showing very good agreement.

## I. INTRODUCTION

Many authors have presented analyses of the coupling between dielectric resonators and microstrip lines [1]-[3]. However all these analyses are limited to  $TE$  mode. In addition to  $TE$  mode, dielectric resonators can also support  $TM$  and  $HE$  (hybrid) modes [4][5]. When dielectric resonators are used in MIC, the coupling to microstrip lines results from both the magnetic field and the electric field of the microstrip line, which will excite all types of the dielectric resonator modes. Quantitative computation of the coupling between a microstrip line and a dielectric resonator for hybrid modes, has not previously reported in the literature.

This paper presents an approximate approach to model both the magnetic and the electric couplings by using lumped element equivalent circuits. The analysis given in this paper shows that the magnetic coupling and the electric coupling can be treated separately. Two degenerate modes are excited, one is by magnetic coupling, the other is by electric coupling. These two degenerate modes have the same resonant frequency. The external

$Q$  [6] of the system composed of the microstrip line and the resonator is computed and the results are compared

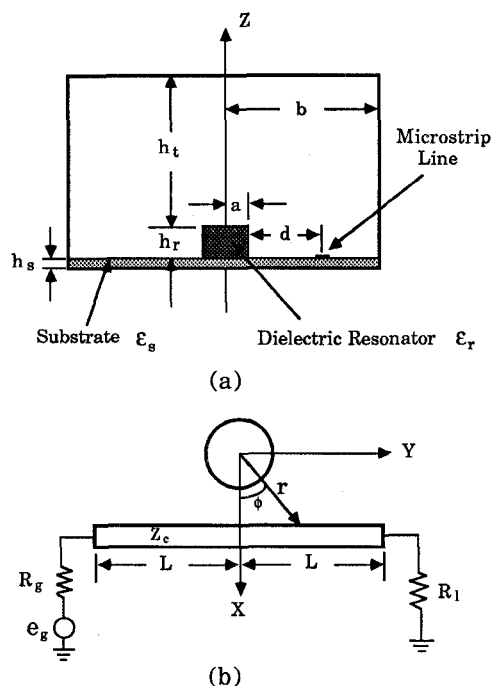


Fig. 1 Configuration

with the measured data, showing very good agreement between theory and experiment.

## II. MODELING

A cross section and a top view of the configuration under consideration (without enclosure) are shown in Fig. 1. The dielectric resonator is placed on the substrate, adjacent to the microstrip line.

The basic assumptions used in the following analysis are:

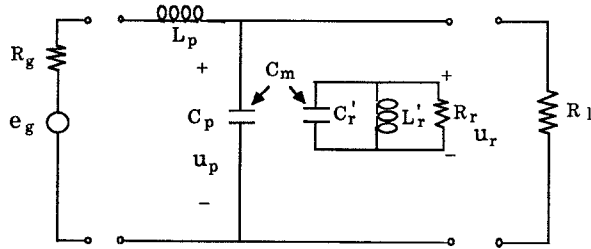
- TEM mode is propagating in microstrip line;
- The microstrip line effect is considered as a small perturbation to the field distribution inside the dielectric resonator, and its effects are neglected;
- The dielectric constant of dielectric resonator is larger than that of the substrate;
- All losses involved are small enough to be neglected;
- The termination of the line and the RF source are matched with the line.

### 1. Electric Coupling

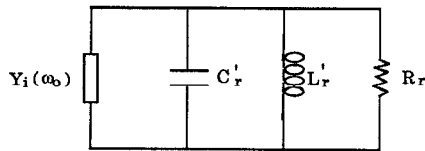
A low-frequency equivalent circuit of Fig. 1 for electric coupling is shown in Fig. 2-a. The coupling between the line and the resonator is characterized by a mutual capacitance  $C_m$ .  $[L'_r, C'_r, R_r]$  and  $[L_p, C_p]$  represent the equivalent parameters of the resonator and of the line, respectively.  $R_l$  is the termination resistance of the line, and  $R_g$  the internal resistance of the source.

The admittance  $Y_i$  induced by  $C_m$  in the resonator, shown in Fig. 2-b, is

$$Y_i(\omega) = \frac{\omega^2 C_m^2}{1/R_l + j\omega C_p + 1/(R_g + j\omega L_p)} \cong \frac{\omega^2 C_m^2 Z_c}{2} \quad (1)$$



(a)



(b)

Fig. 2 Low-frequency equivalent circuit for electric coupling

The external Q is:

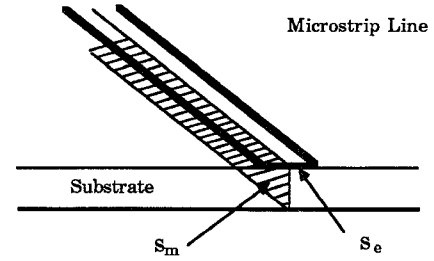
$$Q_{ee} = \frac{\omega_0 C'_r}{Y_i(\omega_0)} = \frac{2}{\omega_0 Z_c} \frac{C'_r}{C_m^2} \quad (2)$$

From Fig. 2-a, the current in the line induced by the voltage  $u_r$  is

$$i = j\omega C_m u_r \quad (3)$$

This current can also be computed from the electric flux through the area  $S_e$  (Fig. 3).

$$i = j\omega \epsilon_s \int_{s_e} \vec{E} \cdot d\vec{s}_e \quad (4)$$



$S_m$  — The cross section in substrate along microstrip line  
 $S_e$  — The area underneath the microstrip line and on the surface of the substrate

Fig.3 The areas where the magnetic flux and electric flux are considered

The electric energy stored in the dielectric resonator is given by

$$w_e = \frac{1}{4} C'_r u_r^2 \quad (5)$$

Combining (3)-(5) and substituting into (2), we have

$$Q_{ee} = \frac{2\omega_0 w_e}{P_{eL}} \quad (6)$$

where

$$w_e = \frac{1}{4} \epsilon \int_V \vec{E} \cdot \vec{E}^* dV \quad (7)$$

$$P_{eL} = \frac{1}{4} Z_c \omega_0^2 \epsilon_s^2 \left[ \int_{s_e} \vec{E} \cdot d\vec{s}_e \right]^2 \quad (8)$$

Where the field  $\vec{E}$  is the electric field of the dielectric resonator.

## 2. Magnetic Coupling

A low-frequency equivalent circuit [1][2] of the structure in Fig. 1 for the magnetic coupling is shown in Fig. 4-a. The coupling between the line and the resonator is characterized by a mutual inductance  $L_m$ .

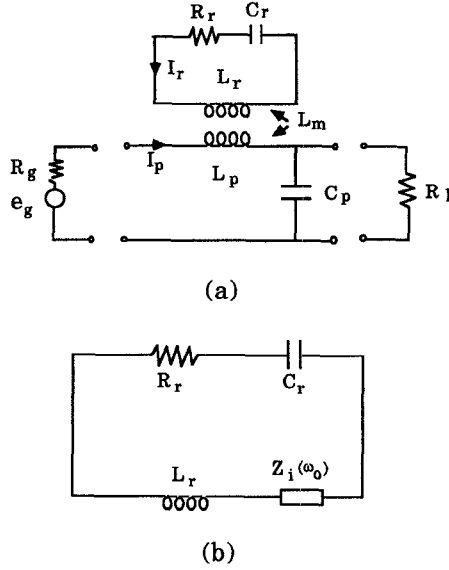


Fig. 4 Low-frequency equivalent circuit for magnetic coupling

The impedance  $Z_i$  induced by  $L_m$  in the resonator is shown in Fig. 4-b. It is known that the external Q is given by [1][2]:

$$Q_{em} = \frac{2\omega_0 w_m}{P_{mL}} \quad (9)$$

where

$$w_m = \frac{1}{4}\mu_0 \int_V \bar{H} \cdot \bar{H}^* dV \quad (10)$$

$$P_{mL} = \frac{\omega_0^2 \mu_0^2}{4Z_c} \left[ \int_{S_m} \bar{H} \cdot d\bar{s}_m \right]^2 \quad (11)$$

Where the field  $\bar{H}$  is the magnetic field of dielectric resonator, and the section  $S_m$  in the substrate is shown in Fig. 3.

## 3. The Relationship between Magnetic and Electric Couplings

The X-component of the magnetic field of the resonator excited by the magnetic field of the TEM mode

propagating in the microstrip line will be maximum at  $\phi = 0$  (in Fig. 1-b), where  $H_x = H_r$ . The functional variation of the hybrid mode fields is expressible in the form [4][5]:

$$H_{rm} \propto \sin(n\phi + \pi/2) \sum_i f_i(r, z) \quad (12)$$

and

$$H_{\phi m} \propto \cos(n\phi + \pi/2) \sum_i g_i(r, z) \quad (13)$$

$$E_{zm} \propto \cos(n\phi + \pi/2) \sum_i k_i(r, z) \quad (14)$$

Where  $f_i$ ,  $g_i$  and  $k_i$  are function of  $r$  and  $z$ .

Substituting (14) into (8), yields:

$$P_{eL} = 0 \quad (15)$$

That means the mode of the resonator excited by the magnetic field of the line does not have any electric coupling to the line.

The Z-component of the resonator's electric field perpendicular to X-Y plane excited by the electric field of the line will be maximum at  $\phi = 0$  (in Fig. 1-b). Then

$$E_{ze} \propto \cos(n\phi) \sum_i k_i(r, z) \quad (16)$$

$$H_{re} \propto \sin(n\phi) \sum_i f_i(r, z) \quad (17)$$

$$H_{\phi e} \propto \cos(n\phi) \sum_i g_i(r, z) \quad (18)$$

Substituting (17) and (18) into (11),

$$P_{mL} = 0 \quad (19)$$

i. e. the mode of the resonator excited by the electric field of the line does not have any magnetic coupling with the line.

Equations (12)-(14) and (16)-(18), show that the modes excited by magnetic field and electric field of the line are perpendicular in space. They are degenerate modes with the same resonant frequency.

## 4. The External Quality Factor $Q_e$

The external quality factor  $Q_e$ , produced by both the magnetic coupling and the electric coupling, is expressible in terms of  $Q_{ee}$  and  $Q_{em}$  from (6) and (9) as:

$$Q_e = 2\omega_0 \frac{w_m/Q_{em} + w_e/Q_{ee}}{P_{mL}/Q_{em} + P_{eL}/Q_{ee}} \quad (20)$$

### III. NUMERICAL AND EXPERIMENTAL RESULTS

Computer programs were developed to calculate the external Q for any of the resonator modes. Fig. 5 shows the dependence of the external Q on the distance between the dielectric resonator and the microstrip line for  $TE_{01}$ ,  $HE_{11}$ ,  $HE_{12}$  and  $TM_{02}$  modes. The magnetic coupling exists in  $TE$  and  $HE$  modes, and the electric coupling exists in  $TM$  and  $HE$  modes. The computed and measured data are in very good agreement.

### IV. CONCLUSIONS

A simple model is proposed that represents the coupling between dielectric resonators and microstrip lines, including both magnetic and electric coupling. These two kinds of couplings can be analyzed separately. Two degenerate modes of the dielectric resonator can be excited by magnetic and electric couplings, respectively. Experimental data shows very good agreement with the computed data.

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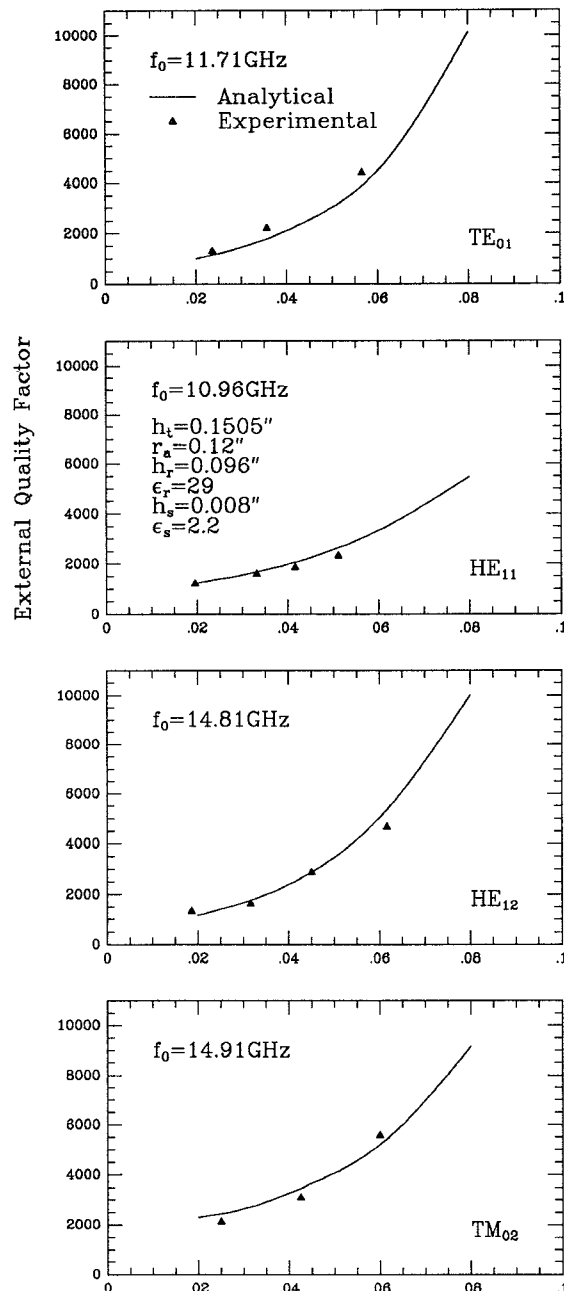


Fig. 5 Distance Between D.R. and Microstrip Line in inch, d